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Rotating Brane Worlds and the Global Rotation of the Universe

Alan H. Guth[1*] and Ali Nayeri[1,2†]

[1] *Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

[2] *Institute for Fundamental Theory, Department of Physics, University of Florida, Gainesville, Florida 32611*

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We introduce a class of brane-world models in which a single brane is embedded in an anti-de Sitter spacetime containing a rotating (Kerr) black hole. In this Letter we consider the case of slow rotation, calculating the metric and dynamics of the brane world to first order in the angular momentum of the black hole. To this order we find that the cosmic fluid on the brane rotates rigidly relative to a Robertson-Walker frame of reference, which in turn rotates rigidly relative to the original Kerr-anti-de Sitter coordinate frame. Corrections to the Friedmann equations and the shape of the brane occur only at higher order. We construct models for which the geometry on the brane is either closed or open, but the open models are described only for small distances from the rotation axis, and may very likely develop pathologies at larger distances.

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In recent years, there has been a great deal of interest in studying the cosmology of universes with extra dimensions [1, 2, 3]. The common feature of all these models is the distinction of the observable universe (the brane world) from the rest of the universe (the bulk). While all the gauge fields with spin less than two are confined to the brane, the gravitons can propagate into the bulk. The cosmology of brane-world models is in general different from standard Friedmann-Robertson-Walker (FRW) cosmology, but if the bulk is anti-de Sitter space (AdS), containing perhaps a Schwarzschild black hole, then the cosmology is very similar to FRW at late times [4]. The presence of a static black hole in the bulk shows up in the modified Friedmann equation as *"dark radiation"* [5, 6, 7]. Though rotating black holes in higher dimensions [8] and with asymptotic AdS behavior in four dimensions [9] were studied many years ago, it was after the advent of the AdS/CFT correspondence [10] that the properties of Kerr-AdS black holes in higher dimensions were studied in great detail [11, 12]. The purpose of this Letter is to investigate the behavior of a brane world in the presence of a bulk Kerr-AdS$_5$ black hole in the slowly rotating regime. In this five-dimensional spacetime the rotating black hole can be characterized by two independent projections of the angular momentum, J_φ, and, J_ψ, but in this Letter we consider only the case in which $J_\psi = 0$, with $J_\varphi \equiv j$. The stationary axisymmetric metric for a five-dimensional single-parameter Kerr black hole can be written as

$$\begin{aligned} ds_5^2 \;=\; & -\frac{\Delta_r}{\rho^2}\left[dT - \frac{k\,j}{\Xi_k}\,S_k^2(\theta)\,d\varphi\right]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \\ & + \frac{\Delta_\theta\,S_k^2(\theta)}{\rho^2}\left[k^2\,j\,dT - \frac{(k^2\,j^2 + r^2)}{\Xi_k}\,d\varphi\right]^2 \\ & + r^2 C_k^2(\theta)\,d\psi^2\,, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \Delta_r &= (k^2\,j_\varphi^2 + r^2)\left(k + \frac{r^2}{\ell^2}\right) - 2m\,, \\ \rho^2 &= r^2 + k^2\,j^2\,C_k^2(\theta)\,, \\ \Xi_k &= 1 - k\,\frac{j^2}{\ell^2}\,, \\ \Delta_\theta &= 1 - k\,\frac{j^2}{\ell^2}C_k^2(\theta)\,, \end{aligned} \tag{2}$$

and

$$\begin{aligned} S_k(\theta) &= \frac{\sin(\sqrt{k}\,\theta)}{\sqrt{k}} \\ C_k(\theta) &= \frac{dS_k(\theta)}{d\theta} = \cos(\sqrt{k}\,\theta)\,. \end{aligned} \tag{3}$$

The parameter $m > 0$ is related to the mass of the black hole, j to the angular momentum for rotation in the φ direction, $\ell^2 = -6/\Lambda_5$ is the AdS$_5$ length scale, where Λ_5 is the cosmological constant, and k is the curvature parameter. A change in $|k|$ with no change in the sign of k can be compensated by a rescaling of r, θ, ψ, T, m, and j, but a change in the sign of k results in a change in the topology of the hypersurfaces of constant r and T.

Note that as $k \to 0$, j completely disappears from the metric, so there is no rotation in this limit. As far as we know there is no solution that behaves smoothly as $k \to 0$ and which preserves a nontrivial effect of j. A $k = 0$ solution with nontrivial j as a distinct case has been described by Klemm [11]. The metric (1) for $k = 1$ was given by Hawking et al. [12].

We will confine our discussion to values of r that are large enough to ensure that $\Delta_r(r) > 0$, and for $k > 0$ we also require $j^2 < \ell^2/k$ to ensure that the metric has the proper signature. With these restrictions, we can show that the spacetime does not contain any closed timelike or null curves. To see this, note that any closed curve can

be parameterized by $X^A(\lambda)$, where $X^A \equiv (T, r, \theta, \varphi, \psi)$ and $\lambda \in [0,1]$ is a parameter, with $dX^A/d\lambda \neq 0$ for all λ. The curve must have a maximal value of $T(\lambda)$, at which $dT/d\lambda = 0$. At this point $(dX^A/d\lambda)^2$ contains only terms proportional to dr^2, $d\theta^2$, $d\psi^2$, and $d\varphi^2$, where the first 3 are manifestly positive. With some algebra the coefficient of $d\varphi^2$ can be written as

$$g_{\varphi\varphi} = \frac{S_k^2(\theta)}{\Xi_k^2 \rho^2}\left[(r^2 + k^2 j^2)\rho^2 \Xi_k + 2mk^2 j^2 S_k^2(\theta)\right] , \quad (4)$$

which is also positive. Thus $(dX^A/d\lambda)^2 > 0$, and therefore the curve cannot be timelike or null.

To study the dynamics of a brane world in this background, we simplify the problem by working perturbatively through first order in the rotation parameter j. In this approximation the metric (1) may be written as

$$\begin{aligned} ds_5^2 \;\approx\; & -F_k(r)\, dT^2 + \frac{dr^2}{F_k(r)} \\ & + 2\, kj\, S_k^2(\theta)\, G_k(r)\, dT\, d\varphi + r^2 d\Omega_3^2 , \end{aligned} \quad (5)$$

where

$$G_k(r) = -\frac{2m}{r^2} + \frac{r^2}{\ell^2} , \quad F_k(r) = G_k(r) + k , \quad (6)$$

and

$$d\Omega_3^2 = d\theta^2 + S_k^2(\theta)\, d\varphi^2 + C_k^2(\theta)\, d\psi^2 . \quad (7)$$

For $k > 0$ this linearization is valid throughout the manifold, for sufficiently small j, but for $k < 0$ the unboundedness of $S_k^2(\theta)$ and $C_k^2(\theta)$ implies that for any j the linearization will break down for sufficiently large θ.

Suppose that a brane is located at $r = r_b(T)$, for some function $r_b(T)$, and that we construct a $\mathbb{Z}_2$-symmetric spacetime consisting of the region $r < r_b(T)$ of the original Kerr-AdS metric (1), with the spacetime for $r > r_b(T)$ replaced by a mirror copy of the spacetime for $r < r_b(T)$. The position of the brane can be conveniently described by $N(X^A) = 0$, where $N = r - r_b(T)$. The spacetime of the brane can be described in 4D coordinates $x^\mu \equiv (t, \theta, \varphi, \psi)$, with an induced metric

$$\begin{aligned} ds_4^2 \;&=\; g_{AB} \frac{\partial X^A}{\partial x^\mu} \frac{\partial X^B}{\partial x^\nu}\, dx^\mu\, dx^\nu \equiv g_{AB}\, Q^A{}_\mu\, Q^B{}_\nu\, dx^\mu\, dx^\nu \\ &\equiv\; \gamma_{\mu\nu}\, dx^\mu\, dx^\nu \\ &=\; -dt^2 + 2\, jk\, S_k^2(\theta)\, G_k(a) \left(\frac{dT}{dt}\right) dt\, d\varphi + a^2\, d\Omega_3^2 , \end{aligned} \quad (8)$$

where

$$dt = \left[F_k(a) - \frac{1}{F_k(a)}\left(\frac{dr_b}{dT}\right)^2\right]^{1/2} dT , \quad (9)$$

and the Robertson-Walker (RW) scale factor is given by $a(t) = r_b\big(T(t)\big)$. Using an overdot to denote a derivative with respect to t, the above relation can be rewritten as

$$\frac{dT}{dt} = \frac{\sqrt{F_k(a) + \dot{a}^2}}{F_k(a)} . \quad (10)$$

To first order in j, the metric (8) can be diagonalized to give a Robertson-Walker metric by introducing a new angular coordinate ϕ defined by

$$\phi(\varphi, t) = \varphi - \int^t \Omega(t')\, dt' , \quad (11)$$

where

$$\Omega(t) = -kj\left[\frac{G_k(a)\sqrt{F_k(a) + \dot{a}^2}}{a^2(t)\, F_k(a)}\right] . \quad (12)$$

$\Omega(t)$ is thus the angular velocity of the RW frame with respect to the Kerr-AdS$_5$ frame. In the RW coordinates the metric is simply

$$ds_4^2 = -dt^2 + a^2(t)\, d\Omega_3'^2 , \quad (13)$$

where

$$d\Omega_3'^2 = d\theta^2 + S_k^2(\theta)\, d\phi^2 + C_k^2(\theta)\, d\psi^2 . \quad (14)$$

We can now study the dynamics of the brane through the junction conditions on the brane. To this aim, one needs to calculate the extrinsic curvature $K_{\mu\nu}$, which is defined in terms of the normalized outward normal vector

$$n_A = \frac{N_{,A}}{\sqrt{g^{BC} N_{,B} N_{,C}}} = -\dot{a}\, \delta_A{}^T + \frac{\sqrt{F_k(a) + \dot{a}^2}}{F_k(a)}\, \delta_A{}^r . \quad (15)$$

(We will use commas to denote coordinate derivatives, and semicolons to denote covariant derivatives.) The extrinsic curvature is defined by

$$K_{\mu\nu} \equiv n_{B;A}\, Q^A{}_\mu\, Q^B{}_\nu , \quad (16)$$

and to first order in j the nonzero components are found to be

$$\begin{aligned} K^t{}_t \;&=\; \frac{a}{\sqrt{F_k(a) + H^2 a^2}}\left(\dot{H} + H^2 + \frac{1}{\ell^2} + \frac{2m}{a^4}\right) , \\ K^t{}_\varphi \;&=\; -4\, kj\, m S_k^2(\theta)/a^3 , \\ K^\varphi{}_t \;&=\; -\frac{kj}{a^3 F_k(a)}\left[a^2 G_k(a) \dot{H} - k\left(\frac{a^2}{\ell^2} + \frac{2m}{a^2}\right)\right] , \\ K^\theta{}_\theta \;&=\; K^\varphi{}_\varphi = K^\psi{}_\psi = \sqrt{F_k(a) + H^2 a^2}/a , \end{aligned} \quad (17)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter on the brane. The extrinsic curvature can be related to the energy momentum tensor $S^{\mu\nu}(x)$ on the brane [15] through the junction condition [13]. Assuming the $\mathbb{Z}_2$ symmetry on

the brane, the junction condition with the sign appropriate to keeping the region $r < r_b(T)$ may be written as

$$S^\mu{}_\nu = \frac{2}{\kappa_5^2}[K^\mu{}_\nu - \delta^\mu{}_\nu K]\,, \tag{18}$$

where $K = \gamma^{\mu\nu}K_{\mu\nu}$ is the trace of the extrinsic curvature. Straightforward calculation leads to

$$\begin{aligned}
K &= \frac{a}{\sqrt{F_k(a)+H^2a^2}} \\
&\qquad \times\left(\dot H + 4H^2 + \frac{4}{\ell^2} - \frac{4m}{a^4} + \frac{3k}{a^2}\right)\,,\\
S^t{}_t &= -\frac{6\sqrt{F_k(a)+H^2a^2}}{\kappa_5^2 a}\,,\\
S^t{}_\varphi &= -\frac{8\,kj\,m}{\kappa_5^2a^3}\,S_k^2(\theta)\,,\\
S^\varphi{}_t &= -\frac{2kj}{\kappa_5^2a^3F_k(a)}\left[a^2\,G_k(a)\,\dot H - k\left(\frac{a^2}{\ell^2}+\frac{2m}{a^2}\right)\right]\,,
\end{aligned} \tag{19}$$

and finally

$$\begin{aligned}
S^\theta{}_\theta &= S^\varphi{}_\varphi = S^\psi{}_\psi = -\frac{2a}{\kappa_5^2\sqrt{F_k(a)+H^2a^2}}\\
&\qquad\times\left(\dot H + 3H^2 + \frac{3}{\ell^2} - \frac{2m}{a^4} + \frac{2k}{a^2}\right)\,.
\end{aligned} \tag{20}$$

Now suppose that $S_{\mu\nu}$ consists of two distinct parts, namely,

$$S^{\mu\nu} = -\lambda\gamma^{\mu\nu} + \tau^{\mu\nu}\,, \tag{21}$$

where λ is the tension of the brane and $\tau^{\mu\nu}$ is the energy-momentum tensor of the matter fluid on the brane. If the matter can be described as a perfect fluid, then

$$\tau^\mu{}_\nu = (\rho_m + p_m)u^\mu\,u_\nu + p_m\,\delta^\mu_\nu\,, \tag{22}$$

in which case $S^\mu{}_\nu$ has one eigenvector with eigenvalue $-(\rho_m+\lambda)$, and three degenerate eigenvectors with eigenvalue $p_m - \lambda$. To linear order in j, $S^\mu{}_\nu$ as shown above has exactly this pattern of eigenvalues, so we can treat it as a perfect fluid. The eigenvalues (to this order) are given by the diagonal elements, so we see immediately that

$$\rho_m + \lambda = -S^t{}_t = \frac{6\sqrt{F_k(a)+H^2a^2}}{\kappa_5^2 a}\,, \tag{23}$$

which with Eq. (6) can be rewritten as a generalized Friedmann equation,

$$H^2 = \frac{1}{3}(8\pi G_N\rho_m + \Lambda_4) + \frac{\kappa_5^4}{36}\rho_m^2 - \frac{k}{a^2} + \frac{2m}{a^4}\,, \tag{24}$$

where $G_N = \lambda\kappa_5^4/(48\pi)$ and $\Lambda_4 = \frac{1}{2}(\Lambda_5 + \frac{1}{6}\kappa_5^4\lambda^2)$. To this order j does not enter the Friedmann equation, so the above equation is the standard one for brane-world cosmology with a Schwarzschild black hole [5, 6, 7]. Similarly $p_m - \lambda$ is the 3-fold degenerate eigenvalue, so $p_m - \lambda = S^\theta{}_\theta = S^\varphi{}_\varphi = S^\psi{}_\psi$ as given in Eq. (20). This expression looks complicated, but with Eq. (23) it implies that

$$\rho_m + p_m = -\frac{2a}{\kappa_5^2\sqrt{F_k(a)+H^2a^2}}\left(\dot H - \frac{k}{a^2} + \frac{4m}{a^4}\right)\,, \tag{25}$$

which by using Eq. (23) again can be shown to be equivalent to the standard energy conservation equation,

$$\dot\rho_m + 3H(\rho_m + p_m) = 0\,. \tag{26}$$

The fluid velocity u^μ is determined by the fact that it is the eigenvector of $S^\mu{}_\nu$ with eigenvalue $S^t{}_t$. Although the eigenvalue is not affected by j to first order, the eigenvector is affected, being given by $u^\theta = u^\psi = 0$, with

$$\begin{aligned}
u^\varphi &= \frac{S^\varphi{}_t}{S^t{}_t - S^\varphi{}_\varphi}\,u^t = \frac{kj\sqrt{F_k(a)+H^2a^2}}{a^2}\\
&\qquad\times\left[\frac{4m}{a^4\dot H - a^2k + 4m} - \frac{G_k(a)}{F_k(a)}\right]\,u^t\,.
\end{aligned} \tag{27}$$

Since $u^\varphi/u^t = d\varphi/dt$ is a function only of t, the above equation describes rigid rotation, in which all points move with the same angular velocity.

Eq. (27) shows that the cosmic fluid is rotating rigidly with respect to the Kerr-AdS$_5$ frame of reference, but observers on the brane would have no way of measuring this quantity. We can, however, transform this result to the RW frame defined by Eq. (11), which implies that

$$\frac{d\phi}{dt} = \frac{d\varphi}{dt} - \Omega(t) = \frac{4\,kj\,m\sqrt{F_k(a)+H^2a^2}}{a^6\left(\dot H - \frac{k}{a^2} + \frac{4m}{a^4}\right)}\,. \tag{28}$$

Since the galaxies would on average be comoving with the cosmic fluid, the angular velocity given above would be directly measurable as an apparent rotation of the distant galaxies relative to the locally inertial (RW) frame of reference.

To observers on the brane, the rotation of the distant galaxies relative to the RW frame would be interpreted as a clear violation not only of Mach's principle, but also the Einstein equations, since in this frame the Einstein tensor is diagonal but the energy-momentum tensor is not. In brane-world cosmology the Einstein equations are in general modified [14] by nonlinear terms and by adding to $8\pi G_N\tau_{\mu\nu}$ the term $-E_{\mu\nu}$, where

$$E_{\mu\nu} \equiv {}^{(5)}C^A{}_{BCD}\,n_A\,n^C\,Q^B{}_\mu\,Q^D{}_\nu\,, \tag{29}$$

and ${}^{(5)}C^A{}_{BCD}$ is the 5-dimensional Weyl tensor. To lowest order in j, the nonzero components of $E_{\mu\nu}$ in the RW

frame (with sign conventions of Ref. [14]) are given by

$$E_{\tau\tau} = \frac{3E_{\theta\theta}}{a^2} = \frac{3E_{\phi\phi}}{a^2 S_k^2(\theta)} = \frac{3E_{\psi\psi}}{a^2 C_k^2(\theta)} = -\frac{6m}{a^4}$$
$$E_{\phi\tau} = E_{\tau\phi} = \frac{8kjm\sqrt{F_k(a)+H^2a^2}}{a^4}\, S_k^2(\theta)\ . \quad (30)$$

Thus, the diagonal entries of $E_{\mu\nu}$ are responsible for the *"dark radiation"*, and the off-diagonal entries are responsible for what might be called *"dark angular momentum"*. In the RW frame this *"dark angular momentum"* cancels the physical angular momentum, so the metric can remain isotropic.

Eq. (28) expresses $d\phi/dt$ in terms of the kinematical quantities H and $\dot{H}$, but we can also express the result in terms of ρ and p. By combining Eqs. (25) and (28), one has

$$\frac{d\phi}{dt} = -\frac{8\,kj\,m}{\kappa_5^2 a^5(\rho_m + p_m)}\,. \quad (31)$$

In this form we can tell how the rotational velocity of the universe would vary with time. The quantity $\rho_m + p_m$ would in most situations be dominated by either radiation or nonrelativistic matter, since a cosmological constant would make no contribution. In the former case $d\phi/dt$ falls off as $1/a$, and in the latter it falls off as $1/a^2$. Thus the rotation would be strongly suppressed by inflation, during which a would grow by many orders of magnitude. Nonetheless, in models with minimal inflation and large initial angular momenta, it is not impossible for the universe to have an observable rotation rate in the present era.

Finally, to verify the properties of the rotation in a manifestly coordinate-invariant way, we calculate the vorticity and the shear of the fluid velocity. The vorticity and the shear are defined in terms of $\theta_{\alpha\beta} \equiv u_{\mu;\nu}q^{\mu}{}_{\alpha}q^{\nu}{}_{\beta}$, where $q_{\mu\nu} \equiv \gamma_{\mu\nu} + u_\mu u_\nu$ is the projection tensor orthogonal to the four-velocity u^μ. The vorticity is the antisymmetric part, $\omega_{\alpha\beta} \equiv \theta_{[\alpha,\beta]}$ and $\omega^2 \equiv \frac{1}{2}\omega_{\alpha\beta}\omega^{\alpha\beta}$. The shear is the traceless symmetric part, $\sigma_{\alpha\beta} \equiv \theta_{(\alpha,\gamma)} - \frac{1}{3}q_{\alpha\beta}\theta^{\gamma}_{\gamma}$, with $\sigma^2 \equiv \frac{1}{2}\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}$. To evaluate these quantities, one first normalizes the velocity vector described in Eq. (27), which to lowest order requires $u^t = 1$. Lowering the index using $\gamma_{\mu\nu}$ then gives $u_\theta = u_\psi = 0$, $u_t = -1$, and

$$u_\varphi = -\frac{8\,kj\,m}{\kappa_5^2\, a^3\,(\rho_m + p_m)} S_k^2(\theta)\,, \quad (32)$$

where we again used Eq. (25). One then finds that the shear vanishes identically, as expected for rigid rotation, and the vorticity has the nonzero components

$$\omega_{\theta\varphi} = -\omega_{\varphi\theta} = \frac{8\,k\,j\,m}{\kappa_5^2\, a^3(\rho_m + p_m)} S_k(\theta)\, C_k(\theta) + \mathcal{O}(j^2)\,, \quad (33)$$

giving

$$\omega^2 = \frac{64 j^2 k^2 m^2}{\kappa_5^4 a^{10}(\rho_m + p_m)^2}\, C_k^2(\theta) + \mathcal{O}(j^3)\,. \quad (34)$$

Note that ω^2 can be written as $\omega^2 = (d\phi/dt)^2 C_k^2(\theta)$, where $d\phi/dt$ was given in Eq. (31). Thus, in the vicinity of the axis of rotation, the vorticity is just the square of the angular velocity relative to the locally inertial frame, as one would expect from Newtonian physics.

The main result of this paper is that a brane-world spacetime with a Kerr-AdS$_5$ black hole in the bulk provides a very simple model of a universe exhibiting global rotation: if the angular momentum is small, the matter in the brane world rotates rigidly relative to the inertial frame of reference. It would be interesting to calculate the implications of such a model for the cosmic background radiation. The rigid rotation is perfectly consistent for a closed brane universe, but for an open brane universe such rigid rotation would lead at large distances (large θ) to spacelike velocities. Our linearized approximation would break down at such large distances, but we do not know if a consistent model can be constructed. These and other issues are under investigation.

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* Electronic address: guth@ctp.mit.edu
† Electronic address: nayeri@MIT.EDU

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